Analysis of Seismic Amplification of Soil to the Preservation of the Archaeological Park of Saqsaywaman – Cusco Peru

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ABSTRACT

The Archaeological Park of Saqsaywaman was officially declared Cultural Heritage of Peru on 11th July 1982 for its historical, archaeological and natural value and for being an inherent part of the ancestral Andean conception and of the current image of the city of Cusco- Perú. In 2009 the walls of the third terrace of Sacsaywaman were damaged. It is assumed that these events were caused by the lateral thrust created by the additional hydrostatic pressure due to rainfall runoff and seismic movements. The problem now, is to preserve the restored walls for posterity. The research assess the dynamic response by means two-dimensional analysis. This analysis determine the seismic behavior of the site in terms of the seismic amplification of the surface ground. This is resulting of effects of topography and stratigraphy. For this purpose, two cross sections of the stratigraphic profile were selected. Non-destructive techniques were used to conduct geophysical surveys applying Multichannel Surface of Wave Analysis (MASW), Seismic Refraction, Microtremor Array Measurement (MAM) methods as well as the Nakamura technique for fundamental period studies. Georadar (GPR) surveys were also performed. Results show that the amplifications within the range of 1.37 to 2.35. The maximum acceleration obtained from the ground varies between 0.30 g and 0.52 g. Also, spectral ratio of the surface register and the rock record was determined to obtain the soil vibration period from 0.45 s to 0.55 s, which was then compared with vibration period obtained from microtremor measurements.

Keywords: seismic amplification, dynamic analysis, period of vibration.

1 INTRODUCTION

The Archaeological Park of Saqsaywaman is considered as Cultural Heritage of Peru. According to INC (2007) its construction began at the end of the XIV century. It is located in the Department of Cusco. In 2009 damage occurred in the walls of the third terrace of the Sacsaywaman, in the central part some walls collapsed, it is assumed that the main cause was the lateral thrust caused by the addition of hydrostatic pressure due to water seepage product of the precipitations (Figure 1). It is also believed that previous earthquakes in 1950 (Mw 6.8), 1986 (Mw 5.2) and 2014 (Mw 5.0) (USGS, 2023) weakened the wall and made it susceptible to collapse.

Peru is located in an area of high seismicity (IGP, 2022). The city of Cusco has the Tambomachay active fault and the Saqsaywaman Archaeological Park has a topography with slopes greater than 20%. It is also known that during an earthquake, the shape of the surface earth can cause a variation in ground motion due to the dispersion and diffraction of seismic waves (Sanchez-Sesma et al, 1982). This variation is known as topographic effects. The study of these effects is important to evaluate the damage caused by earthquakes in mountainous areas, depressions or areas where there is a combination of both forms. Observations since 1902 show that the intensity of damage at the top of hills is much greater than at their base, as has been documented in numerous destructive earthquakes (Geli et al., 1988).

To understand this effect of topography, different theoretical models have been proposed to analyze the propagation of seismic waves in elevated terrain. These first models presented a homogeneous semispace including an isolated hill in 2D, with the horizontal component of *S* waves (SH), P waves and the vertical component of *S* waves (SV). Studies by Sanchez-Sesma et al. (1982) and Trifunac (1973) examined the spatial variation of ground motion in slopes with different geometric shapes (triangular, semi-cylindrical, semi-elliptical, etc.). They emphasize the importance of the relationship between the valley width and the incident SH wavelength, as well as the angle of incidence. The above-mentioned theoretical models of seismic wave propagation in elevated topographies have shown a consistent trend of amplification at the hilltop. Amplification, measured as the ratio of motion at the top to that at the base, has yielded values between 1 and 3 for the horizontal and vertical components, as well as for the transverse SH component, in both the time and frequency domains.

Figure 1. *View of Saqsaywaman Park (left). Collapsed Inca wall (right). (Miksad et al., 2013)*

The research has focused on the evaluation of the two-dimensional dynamic response of the soil inside the Saqsaywaman Archaeological Park, using the finite element method. For the characterization of the soil, non-destructive techniques were used by means of geophysical tests, in order to avoid deterioration of the existing monuments. The purpose of this analysis is to determine the seismic amplification at the ground surface of two cross sections in the study area. This information will be used to evaluate the damage and prevent future collapse of the existing walls in the event of seismic events and to propose alternative solutions.

2 GEOPHYSICAL TESTS

The geophysical investigation was carried out in three stages in the months of July 2015, June 2016 and July 2017, in these three stages the following tests were performed: 62 Seismic Refraction lines to determine the P-wave velocity (Vp) of the ground, 16 MASW-1D tests and 06 MASW-2D tests, 11 MAM (Microtremor Array) tests to determine the S-wave velocity (Vs); 373 Georadar (GPR) lines to estimate the soil type and presence of anomalies and 64 microtremor points were measured to estimate the fundamental period of the ground. These microtremors were placed in the form of a grid with a variable spacing of 20 and 100 m. A summary of the number of tests and their length is shown in Table 1.

The location of the tests was defined according to the topography, geology, the disposition of Inca wall zones and the free area available in the Saqsaywaman, allowing to cover the entire study area, as shown in Figures 2 to 4

Table 1. *Summary of test executed, in three stages*

Date	Seismic Refraction		MASW		MAM		Georadar		Microtremor
	N°	**L (m)**	**N°**	**L (m)**	**N°**	**L (m)**	**N°**	**L (m)**	**N°**
2015-07(1st stage)	32	977	4	218	1	100	135	2239	-
2016-06(2nd stage)	30	1080	12	-	-	-	25	838	-
2016-07(3rd stage)	-	-	6	255	10	1000	213	5788	64
	62	2057	22	473	11	1100	373	8865	64

N°= Number of field measurements

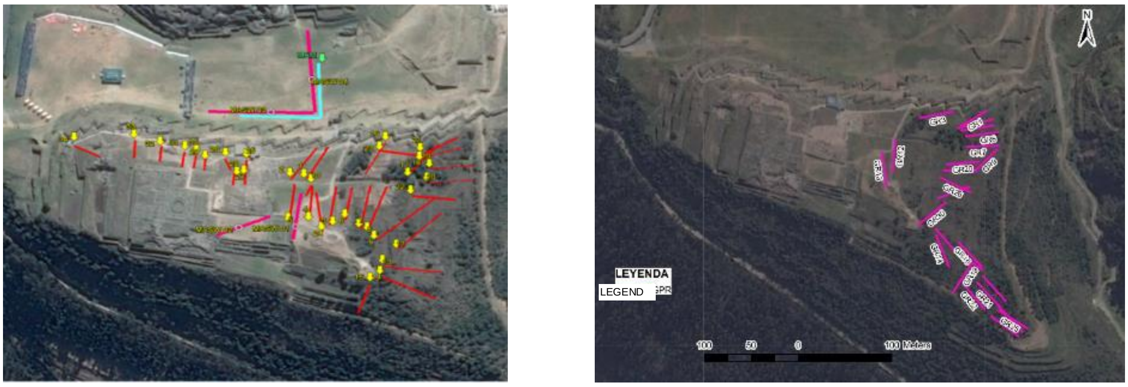


Figure 2. *Plan view of geophysical test carried out on first stage (left). View of GPR test on the second stage (right)*

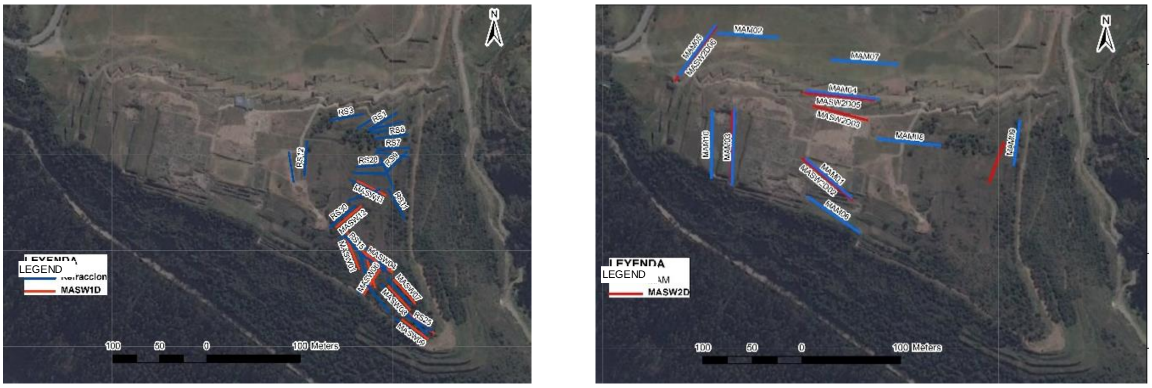


Figure 3. *View of Seismic Refraction and MASW surveys on the second stage (left). View of MAM and MASW surveys on the third stage (right)*

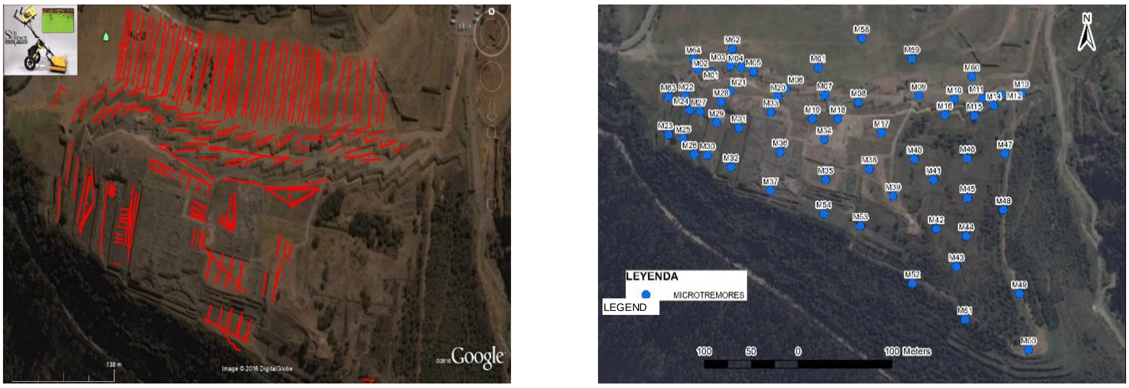


Figure 4. *View of GPR surveys on the third stage (left) . View of Microtremor surveys on the third stage (right).*

3 INTERPRETATION OF GEOPHYSICAL TESTS

3.1 Seismic Refraction

Seismic Refraction measurement is linear array could be located on sloping ground and flat ground. The results obtained are the two-dimensional profiles of P-wave velocity (Vp), of these seismic profiles, the minimum and maximum depth of investigation was 3.0 m and 15.0 m respectively. The obtained values of Vp vary from 210 m/s to 2300 m/s. Figure 5 shows the location of array L29, L30 and L31, and the Vp profile obtained for array L30.

Figure 5. *Location of L29, L30 and L31 seismic refraction lines (left). P-wave profile of the L30 (right)*

3.2 MASW

MASW measurement is linear array which is located to perpendicular to slope line of sloping ground also this array was located in flat ground. The results obtained are one-dimensional and two-dimensional shear wave velocity (Vs) profile for MASW1D y MASW2D test, respectively. For MASW1D the results shows the minimum and maximum values of Vs is 220 m/s and 616 m/s, respectively. For MASW 2D shows the test was determined that the minimum shear velocity is 193 m/s, and the maximum value is 597 m/s. These values allows to infer that the ground has a relative density from loose to dense, according to the range of value Vs in the standard ASCE 7-16. Figure 6 shows corresponding to one-dimensional Vs profile of LS30 line and two-dimensional Vs profile of LS2D array.

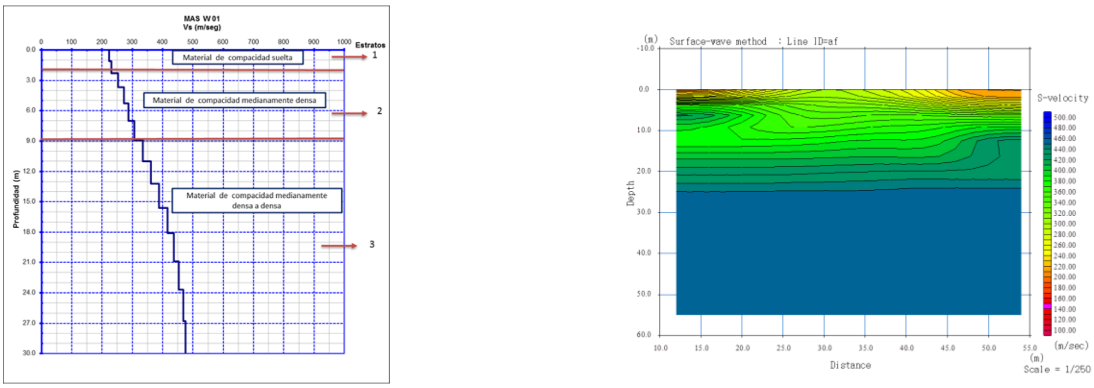


Figure 6. *One-dimensional Vs profile of the LS30 line obtained of MASW1D. (left). Two-dimensional Vs profile of the LS2D array, obtained of MASW2D test (right)*

3.3 MAM

The results obtained from the MAM test correspond to profiles of shear wave velocities (Vs), up to a depth of 100 m. The maximum values of Vs reached were 950 m/s. Figure 7 shows the dispersion curve and its corresponding Vs profile of the MAM test located at the top of terrace. It is observed that the Vs values increase up to 14 m, then the shear wave velocity decreases up to 26 m likely due to change of material, after the value of the Vs increases up to 80 m.

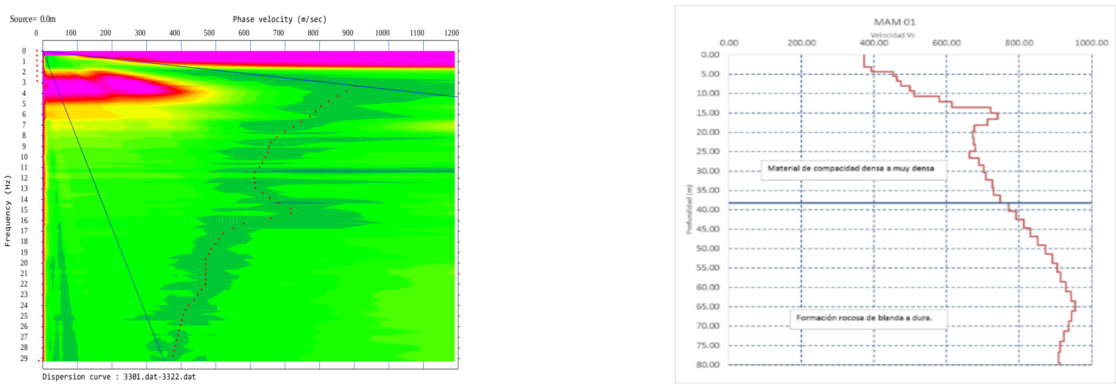


Figure 7. *Dispersion Curve from MAM test (left). Vs profile obtained from the MAM test (right)*

3.4 Georadar (GPR)

According to the results of the radargrams, in general terms, it has been possible to identify three types of materials such as clays with rock fragments, with variable thicknesses of 0.5 m to 3.0 m, clays with gravel and clay with sand with varying thickness of 2.0 m to 4.0 m. In some radargrams the presence of isolated rock fragments was found, which may be part of the buried Inca walls. Figure 8 shows the location of the lines with GPR in flat area and the anomalies in the form of a fringe that can be part of buried Inca walls.

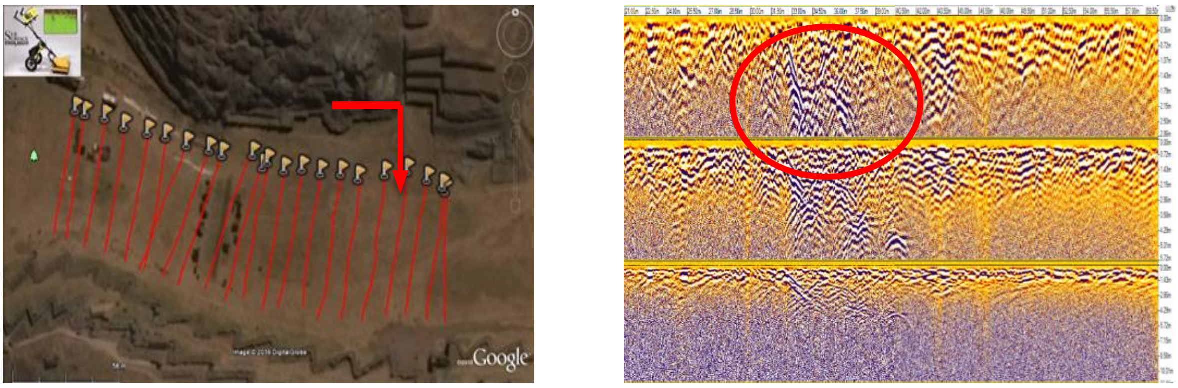


Figure 8. *Location of GPR Lines in flat area, arrow point out the L-5 line (left). Radargram of L-5 line shows anomalies in the form of a fringe (right)*

3.5 Microtremor HV

Due to the lack of information regarding vibration periods in the study area, microtremors were measured in 64 points distributed Saqsaywaman area. The vibration periods were obtained by the ratio between of the Fourier amplitude spectra of the horizontal and vertical motion (Nakamura, 1989), the spectra was smoothed (Kono & Omachi, 1998). The fundamental vibration periods were obtained, which vary from 0.35 s to 0.57 s. In Figure 9, three zones can be appreciated, the green color represents vibration periods less than 0.4 s. The yellow color represents vibration periods within the range of 0.4 s to 0.5 s, covering most of the Saqsaywaman area. The red color represents periods greater than or equal to 0.5 s.

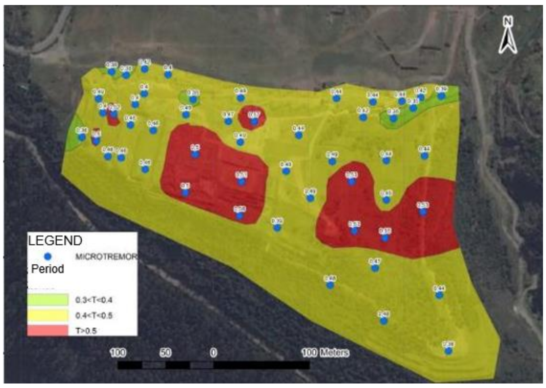


Figure 9. *Isoperiods map.*

4 TWO-DIMENSIONAL DYNAMICS ANALYSIS

In order to perform the dynamic analysis, it is necessary to know the stress-strain behavior of the layers (G/Gmax and damping ratio curves), geometric conditions (topography and depth) and seismic conditions (time-history, PGA rock). Since there were no data on historical earthquakes, the seismic hazard was evaluated and a synthetic accelerogram was generated.

4.1 Seismic Hazard and Synthetic Accelerogram

For the evaluation of the Seismic Hazard, with a probabilistic method, we used the subduction and continental seismic sources and the recurrence parameters established by Gamarra (2009), and the rock spectral acceleration attenuation laws for subduction earthquakes and earthquakes Continental of Young et al. (1997) and Sadigh, et. al. (1997) respectively. As a result, the uniform hazard spectrum (UHS) was obtained for a return period of 475 years and a probability of 10% being exceeded in the exposure period of 50 years. The UHS was obtained by software Crisis 2015 (Ordaz et al.)

Due to there is no data of strong ground motion in Cusco, an accelerogram compatible to uniform hazard spectrum was generated by means the spectral matching method (Hancock et al., 2006). This method modify an seed record in time domain analysis to fitting the UHS. The accelerogram used was from the Paruro earthquake on September 27, 2014, which was recorded at the Tambomachay station. This modified record was performed with SeismoMatch software. Figure 10 shows the uniform hazard spectrum and matched spectrum from accelerogram, and compatible accelerogram its the maximum acceleration is 0.21 g

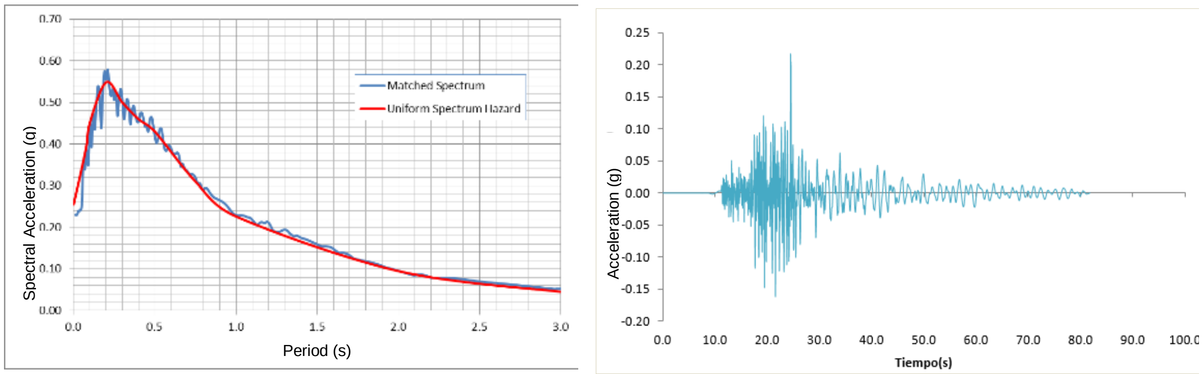
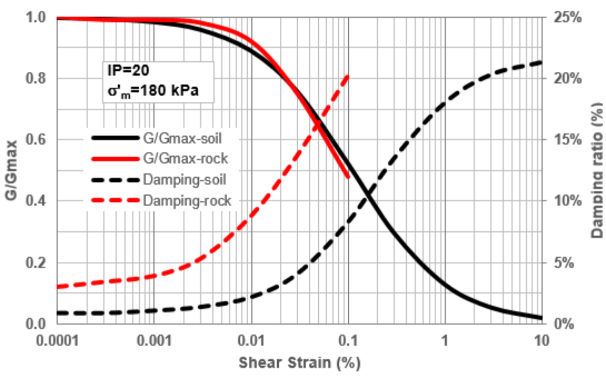


Figure 10. *Uniform hazard spectrum and matched spectrum of synthetic accelerogram (left). Synthetic Accelerogram (right)*

4.2 Non-linear behavior of soil

The empirical equations proposed by Zhang et al (2005) were used for the nonlinear behavior of the soil. For the rock layer, the modulus reduction and damping curve proposed by Schnabel et al (1975) was used. For the base layer, the linear elastic model with a damping of 3% was considered. Figure 11 shows the G/Gmax and damping ratio for soil and rock.



Proposed Relationship by Zhang et al (2005)

Shear modulus

$G/G_{max} = 1/\left[1+(\gamma/\gamma_r)^{\alpha}\right]$

$\gamma_r = \gamma_{rm1}(\sigma'_m/100)^{-k}$

$\gamma_{rm1} = 0.001PI + 0.0749$

$k = 0.316e^{-0.0142PI}$

$\alpha = 0.0021PI + 0.034$

PI=Plastic index

σ'm=mean efective confining stress

Damping ratio

$D_{m1} = 0.008\text{PI} + 0.82$

$D_m = D_{m1}(\sigma'_m/100)^{-k/2}$

$D = 10.6\,(G/G_{max})^2 - 31.6(G/G_{max}) + 21 + D_m$

Figure 11. *Modulus reduction G/Gmax and damping ratios for soil and rock*

4.3 Dynamic Analysis

The two-dimensional dynamic analysis was performed using the Equivalent Linear (EL) Method (Seequent, 2022). For two cross-sections A-A and B-B the analysis was carried out. Sections A-A and B-B are conformed for three layers, the first and second layer was consider as nonlinear material and the third layer was considered as linear material. The first layer is medium to dense material and Vs values are from 180 m/s up to 360 m/s. The second layer is a dense to very dense material and Vs varies in the range of Vs 360 m/s to 760 m/s. The third layer is rock (bedrock) which has an value of Vs=760 m/s (Figure 12).

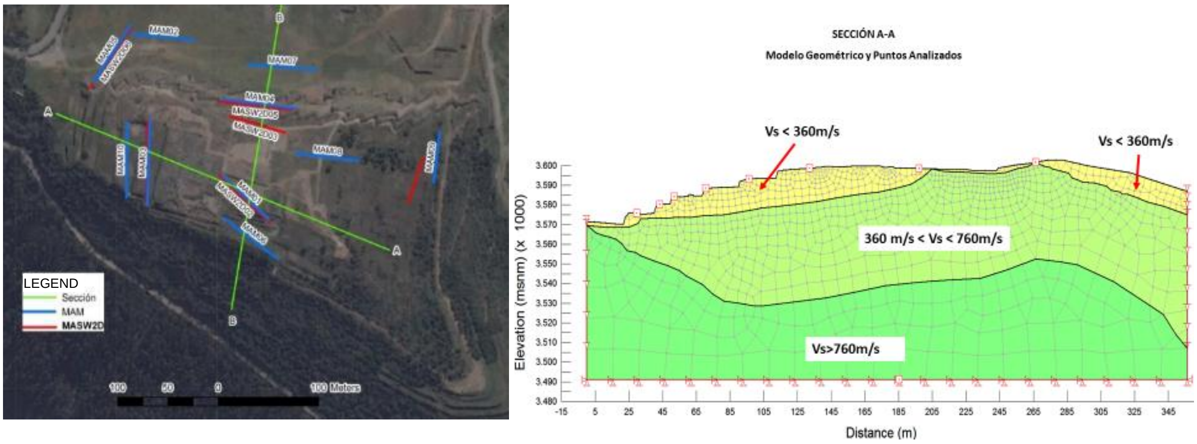


Figure 12. *Location of cross-sections for two-dimensional analysis (left). Cross-section A-A (right)*

Likewise, in each section, the response was evaluated at nine points on the surface. The results show the maximum strain was 3.5 x10^{-3}% .It ocurred at depth 10 m, which was considered appropriate using EL analysis, also were obtained: maximum accelerations, maximum velocities, maximum displacements and acceleration spectra at the ground surface. Then, the amplification of the maximum ground acceleration with respect to the input motion was determined and the fundamental vibration period (at large deformations). This was estimated by the ratio of the acceleration spectrum at the surface and the acceleration spectrum of the input motion record. Table 2 shows the results of the accelerations, velocities, displacements, the amplification ratio (AR) and the period of vibration in sections A-A and B-B.

Table 2. *Accelerations (A), velocities (V), displacements (D), amplification ratio (AR) and period of vibration (T) for the cross-sections A-A and B-B*

Profile A-A						Profile B-B					
Point	**A (g)**	**V (cm/s)**	**D (cm)**	**AR (-)**	**T (s)**	**point**	**A (g)**	**V (cm/s)**	**D (cm)**	**AR (-)**	**T (s)**
P-1	0.32	17.1	1.6	1.44	0.45	p-a	0.30	18.2	1.8	1.37	0.55
p-2	0.41	23.2	2.1	1.84	0.45	p-b	0.42	20.0	2.1	1.86	0.55
p-3	0.41	25.4	2.3	1.83	0.47	p-c	0.38	22.5	2.3	1.71	0.55
p-4	0.40	27.6	2.5	1.80	0.45	p-d	0.41	27.1	2.5	1.85	0.55
p-5	0.45	30.7	2.6	2.00	0.55	p-e	0.49	31.7	2.8	2.18	0.55
p-6	0.45	30.4	2.7	2.00	0.55	p-f	0.50	32.5	2.9	2.24	0.55
p-7	0.38	22.3	2.5	1.72	0.55	p-g	0.52	33.0	2.9	2.35	0.55
p-8	0.39	27.0	2.8	1.73	0.55	p-h	0.44	28.1	2.7	2.00	0.55
p-9	0.45	28.1	2.8	2.00	0.55	p-i	0.43	22.6	2.5	1.95	0.55

4.4 Validation of Two-dimensional Dynamic Analysis

For the validation of obtained results, the spectral ratio of the two-dimensional analysis and the microtremor were compared. The first consists of obtaining the ratio of the Fourier amplitude of the motion at the ground surface with respect to the Fourier amplitude of the motion in the rock, from this spectrum obtain the fundamental vibration period, this spectrum is called HV-Calculated. The second one is to obtain the vibration period from the microtremor measurement as described in section 3.5, this spectrum is called HV-mean. The comparison between the spectra HV-calculated with HV-mean spectra shows that the amplitudes of HV-calculated are smaller than the amplitudes of HV-mean (Figures 13 and 14) and the fundamental period of the calculated-HV spectrum is slightly larger than the fundamental periods of the HV-mean spectrum (Table 3), which is due to the fact that during a strong earthquake the stiffness behavior of the soil is reduced and the damping increases, so the material behaves slightly more flexible.

4.5 Analysis and Discussion of Results

From the results obtained, it is observed that in section A-A, the greatest response in terms of acceleration occurs in the highest part, specifically at points 5, 6 and 9. The maximum acceleration obtained was 0.45 g, which is higher than the acceleration obtained in rock (0.22 g). Considering the topographic effect and stratigraphy, a soil amplification of 2.0 was obtained. Additionally, the fundamental period of vibration on this point was evaluated, which is 0.55 s. Likewise, in this section the peak ground motion accelerations at the surface vary from 0.32 g to 0.45 g, and the amplification of 1.44 to 2.00. The fundamental periods of vibration vary from 0.45 s to 0.55 s (Figure 13). It is also observed that the maximum velocity values are within the range of 17 cm/s to 30 cm/s and the maximum displacements from 1.5 cm to 2.8 cm.

On the other hand, in section B-B, the highest response in terms of acceleration occurs at the highest part, at points "d", "e", "f" and "g". The maximum acceleration at point "g" is 0.52 g, while points "d", "e" and "f" are 0.41 g, 0.49 g and 0.50 g respectively. At these points the accelerations are higher than the acceleration in the rock. Considering the conditions of topography and soil type the soil amplifications were 1.85, 2.18, 2.24 and 2.35 respectively. In addition, the fundamental period of vibration was evaluated. In this section the range of surface accelerations varies from 0.30 g to 0.52 g, and the amplification of 1.37 to 2.35 and the fundamental period of vibration is 0.55 s (Figure 14). In addition, it is observed that the maximum velocity values are within the range of 18.2 cm/s to 33.0 cm/s and the maximum displacements from 1.8 cm to 2.9 cm.

It is also observed that when comparing both sections, it can be seen that the maximum acceleration values obtained on the surface and the amplification values are similar.

Table 3. *Comparison of calculated and measured predominant period for cross-sections A-A and B-B*

Profile A-A			Profile B-B		
Point	T[1] (s)	T[2](s)	Point	T[1] (s)	T[2](s)
P-1	0.45	0.40	p-a	0.55	*
p-2	0.45	0.42	p-b	0.55	*
p-3	0.47	0.45	p-c	0.55	0.55
p-4	0.45	0.43	p-d	0.55	*
p-5	0.55	0.47	p-e	0.55	0.55
p-6	0.55	0.42	p-f	0.55	*
p-7	0.55	0.50	p-g	0.55	0.55
p-8	0.55	0.50	p-h	0.55	0.50
p-9	0.55	0.50	p-i	0.55	0.50

[1] Obtained from Two-Dimension analysis   [2] Obtained from Microtremor
* No measured data

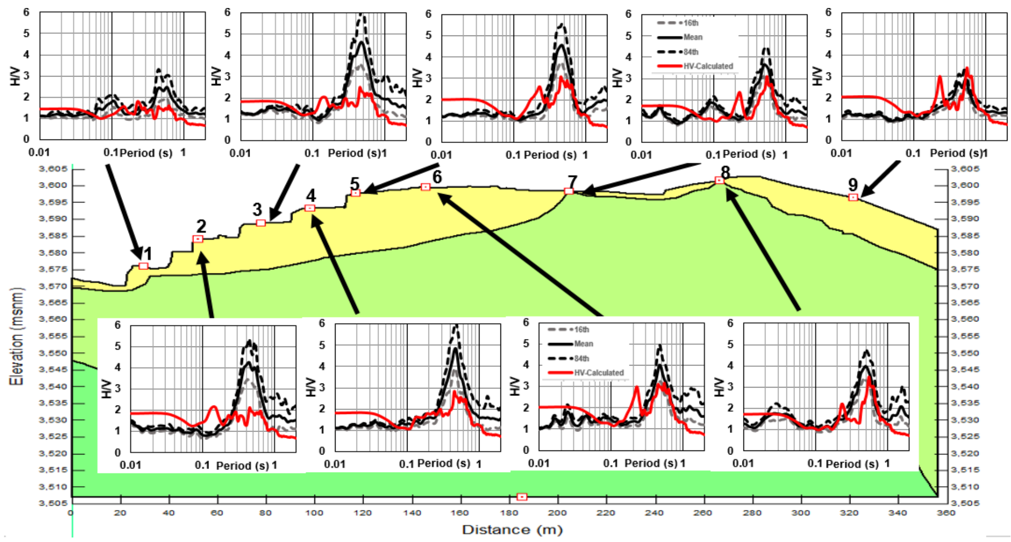


Figure 13. *Comparison de spectral ratio of cross-section of A-A*

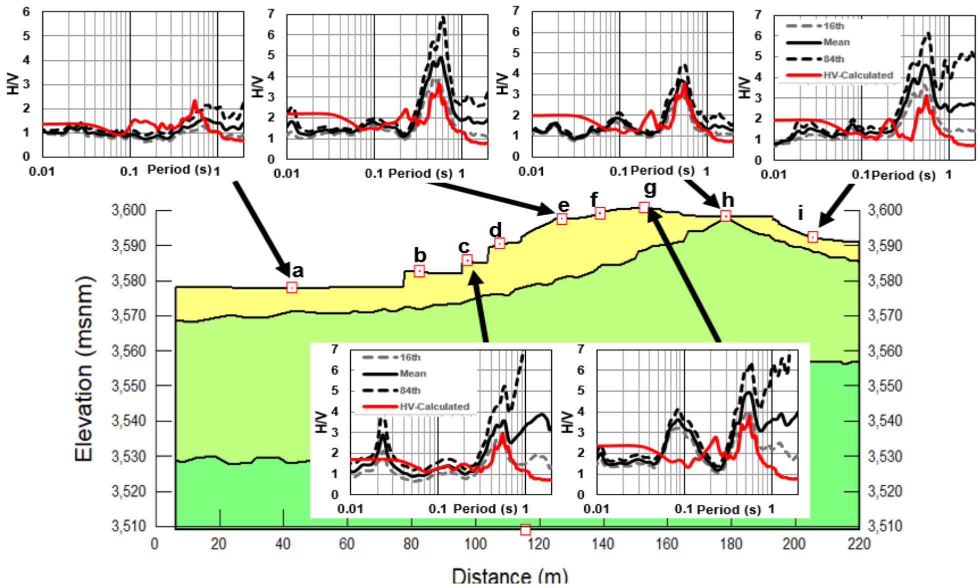


Figure 14. *Comparison de spectral ratio of cross-section of B-B*

4.6 Effects of the Inca Wall

In order to show the effect of largest earthquake associated with the previously calculated accelerations, a stability analysis of the Inca walls has been carried out, considering the geometry of the platforms and the dimensions of the rocks. According to what was observed, in the back of the stone wall as show in Figure 1, there is gravel with oversizes (φ'= 36° and C'=10 kPa) . It is known that a section of the wall collapsed when the drainage system did not work, that is, the soil became saturated during the rains and generated hydrostatic pressures that could not be supported by the wall in that section. Performing a back-analysis, it was possible to define that, for the unsaturated condition, the resistance parameters adopted are adequate. If we now carry out a seismic analysis (pseudostatic) with, the overturning safety factor is reduced to values less than unity. (Figure 15)

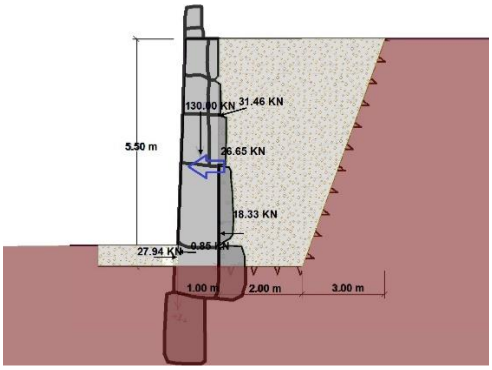


Figure 15. *Pseudostatic analysis, FS less than 1.*

5 CONCLUSIONS

- The combination of the different indirect exploration methods (geophysical) allowed to accurately determine the elastic parameters of the soil, and the combination of the MASW and MAM test allowed to obtain the Vs wave velocity profile at the depth of 100 m.
- The results shows the amplifications of soil within the range of 1.37 to 2.35, ,it due to the topographic and the stratigraphy effects of the site generated amplifications. From the two-dimensional dynamic analysis, it was obtained that the maximum acceleration at the surface varies from 0.30 g to 0.52 g. The fundamental period of vibration obtained from the two-dimensional analysis varies from 0.45 s to 0.55 s. The vibration period obtained by means of microtremors is between 0.40 s and 0.50 s.
- The results shows that if a strong earthquake occurs, the effect on the Inca walls will be devastating.

- This information will allow us to analyze the damage caused to the Inca walls, propose alternative solutions and simulate the behavior of the existing walls in the presence of a seismic event for the preservation of the Park of Saqsaywaman.

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